

Study of composite polymer degradation for high pressure hydrogen vessel by machine learning approach

K. Kadri^a, A. Kallel^c, G. Guerard^b, A. Ben Abdallah^a, S. Ballut^a, J. Fitoussi^c, M. Shirinbayan^{c,*}

^a*R&D Departement, Aptiskills, 28 Rue Edouard Vaillant, 92300 Levallois-Perret, France*

^b*De Vinci Research Center, La Défense, Léonard de Vinci Pôle Universitaire, Paris, France*

^c*Laboratoire PIMM, Arts et Métiers, CNRS, Hesam Université, 151 boulevard de l'Hôpital, Paris, France*

Abstract

The aim of this paper is to study the degradation of a composite material under static pressure. The high pressure condition is similar to the one encountered inside hydrogen tanks. Damage modeling was used to evaluate the behavior of hydrogen tanks to high pressure. A practical approach, coupling a finite element method (FEM) simulation and machine learning algorithm (ML), is suggested. The representative volume element (RVE) was used in association with a choice of a behavior law and a damage law as an input data. Algorithms for ML classification such as K-nearest neighbors (k-NN) and a special k-NN with a dynamic time warping (DTW) metric were used. The hierarchical clustering through dendrograms visualizations allowed to exhibit the impact of composite parameters in relation to fiber, matrix properties and fiber volume fraction on the strain degradation under external static pressure. Continuing this, the optimum RVE which shows a low degradation value will be exhibited.

Keywords: Hydrogen pressure vessel, RVE, Damage law, Behaviour law, Feature classification, Time series, Machine learning, Classification, Dendrogram.

*Corresponding author, Mohammadali.SHIRINBAYAN@ensam.eu

1. Introduction

Composite materials made of a polymeric matrix and a fiber as a reinforcement can offer remarkable physical and chemical properties, such as high strength, low specific weight, fatigue and corrosion resistance, and low thermal expansion. These physical properties enable composites to develop new engineering applications. In fact, they are in a pole position candidate for the type 4 hydrogen pressure vessels. They are promising materials for this application due to the broad scope engineering architecture possibilities. By designing the micro-structure, the composite properties can be controlled and functionally upgraded. Homogenization method [1, 2, 3] will be used to study the structure of the composite material and its properties. This simplification approach [4, 5] of Mean-field calculation is a prediction of global behaviour when the microstructure is heterogeneous. This method offers advantages like evaluation of composites such as structure, electromagnetic, and thermal properties! as well as the study flow in porous media [6]. In a nutshell, this method can be described as the calculus of the stresses and strains at any point of the homogenized medium. They are noted S and E and are assumed to be the over all average of any RVE of the corresponding local microscopic quantities σ and ϵ . We use the same notation as in this article [7], and we are reminded how S and E are calculated within the homogenization method.

$$S = \frac{1}{|V|} \int_{\partial V} dS x \otimes \underline{\underline{\sigma}} \cdot \underline{n} \quad (1)$$

$$E = \frac{1}{|V|} \int_{\partial V} dS \frac{1}{2} (\underline{u} \otimes \underline{n} + \underline{n} \otimes \underline{u}) \quad (2)$$

Here, x is a point of the RVE and its boundary, u the microscopic displacement field, n the normal vector to the closure ∂V and the term $\underline{\underline{\sigma}} \cdot \underline{n}$ is the boundary traction. The symbol \otimes denotes dyadic tensor product. The previous equations 2 allow a simplified computation of the macroscopic variables in comparison with volume integrals over the RVE. For a periodic media, the microscopic fields must have suitable periodicity conditions ensuring continuity of boundary tractions and displacements across two opposite faces of the cell. $\underline{\underline{\sigma}} \cdot \underline{n}$ must take equal and opposite values at two corresponding points of ∂V . For this reason we will use periodic boundary conditions

31 (PBC) that will be explained in the following.

32

33 Metals as well as composites have been studied through the scope of
34 ML and deep learning (DL) in term of properties and fatigue life. Studies
35 have been conducted to explore the contribution of ML to solve the issue of
36 properties prediction [8] and defect location, size, morphology in life cycle of
37 a metal Ti-6Al-4 V alloy [9]. Support Vector Machines (*SVMs*) algorithm
38 was used to accelerate the optimization process. The grid search approach
39 with cross validation was selected for fitting the model parameters. The
40 predicted fatigue life can reach up to 0.99 precision only by learning from
41 defects. An other major axe of research was to employ ML and DL to
42 detect cracks in pressure vessels at an early stage [10]. Crack identification
43 techniques for pressure vessels that used genetic algorithm (GA)-based on
44 feature selection, deep neural network (DNN) on acoustic emission (AE)
45 examination. Heterogeneous features like time domain, frequency domain,
46 and time-frequency were selected by GA to represent the most discriminant
47 features that are highly effective for the DNN classifier when identifying crack
48 types.

49 Experimental analyses of composite behaviour are costly and often involve
50 high nonlinearity. ML techniques provide effective models that can capture
51 high nonlinearity [11]. In this previous study, using a finite element modeling
52 as a starting point, different ML techniques like Random Forest Regressor,
53 Decision Tree algorithm, and Multiple Linear Regression were employed to
54 establish the causal relation between the fiber angle and buckling capacity
55 of a cylinder under bending-induced loads.

56 The present study attempts to expand the scope of ML algorithms to the
57 issue of composite damage material as a response to external static pressure.
58 In fact, the importance of understanding the degradation of a composite ma-
59 terial under static pressure is of utmost importance to guarantee the safety
60 of hydrogen storage vessels [12].

61 The coupling between FEM simulation and ML will provide numerical sup-
62 port to develop a well suited composite. Optimal ML algorithms aim to im-
63 prove the predictive performance of the material resistance to high pressure
64 hydrogen tank. It will also shed light on degradation of a composite under
65 different solicitations. The subsequent sections of this article are structured
66 as follows: the first section exhibits the central problem of ductile damage
67 and rupture criteria under high static pressure. The second section presents
68 the modeling methodology. The result of FEM simulation will be exposed

69 in term of ductile damage where the effect of RVE 's size will be addressed.
 70 The third section exposes the use of ML modeling to evaluate the damage
 71 behavior. It will introduce the description of the used database and the
 72 pre-processing treatment. In this supervised learning for classification, the
 73 model evaluation metrics will be discussed. Deployment of both k-NN, and
 74 dynamical time warping k-NN models are described with the performances
 75 and comparison of proposed models. The results in term of material degra-
 76 dation classification and performance will be given. Finally, the last section
 77 draws conclusions and outlines promising directions for future work.

78 2. Problem position

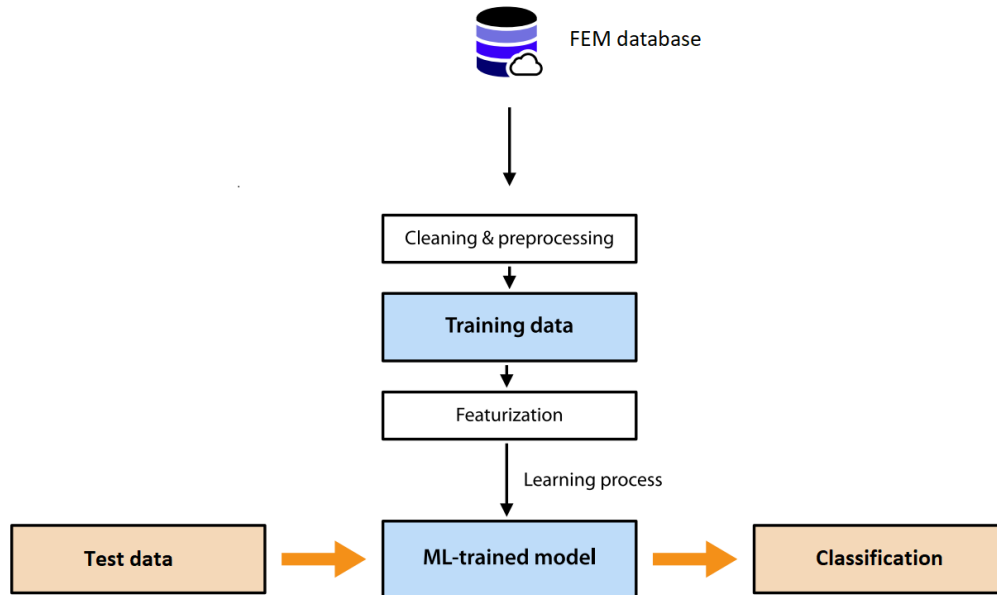


Figure 1: A schematic representation of the key steps of the ML workflow. The composite material at microscopic scale is simulated by FEM. The post-processing of the ABAQUS simulation allows to extract the dataset for ML classification. Classification performance is measured over the test dataset (*adapted from [13]*).

79 The general workflow for the problem studied in this article is illustrated
 80 in Figure 1. The study aims to predict patterns of degradation in compos-
 81 ite materials by leveraging a hybrid approach that combines Finite Element
 82 Method (FEM) simulations and Machine Learning (ML) techniques. The

83 research problem is addressed by conducting FEM simulations of Represent-
84 tive Volume Elements (RVEs) of square unit cell unidirectional composites
85 (UD). A body-centered cubic (bcc) unit cell configuration is employed as it is
86 suitable for representing the fiber-matrix interaction. The aim is to use ML
87 algorithms, specifically K-Nearest Neighbors (KNN) and KNN with Dynam-
88 ical Time Warping (DTW), to classify and visualize the degradation patterns
89 of different composites based on their matrix type, fiber type, and volume
90 fraction. By analyzing the hierarchical clustering tree derived from these
91 algorithms, insights into the relative impacts of matrix composition, fiber
92 type, and volume fraction on the degradation behavior of the composites can
93 be gained. The overarching aim is to develop a predictive framework for un-
94 derstanding and comparing composite degradation mechanisms, facilitating
95 advancements in material design and engineering.

96 *2.1. Damage parameter for uniaxial loading conditions*

97 Damage is characterized by the decline of material stiffness. It plays a
98 central role in the analysis of fiber-reinforced composite materials. Several of
99 these materials exhibit elastic-brittle behavior. Damage in these materials is
100 initiated without significant plastic deformation. The fibers are assumed to
101 be parallel. Material properties are specified in a local coordinate system 1.
102 However, the material response can also be defined in terms of the engineering
103 constants and by specifying the elastic stiffness matrix directly as mentioned
104 in the following section.

105 *2.2. Ductile damage*

106 Investigating the damage process in a composite material under constant
107 loading can be challenging. Many studies on the laws of ruptures have been
108 carried out [14, 15, 16, 17].

109 The ductile damage initiation criterion is a model used to predict the trigger-
110 ing of damage due to nucleation, growth, and coalescence of voids in ductile
111 metals [18, 19, 20, 21]. It has been used also for composites [22, 23, 24]. The
112 model assumes that the equivalent plastic strain at the onset of damage is a
113 function of stress triaxiality and strain rate [25]. The ductile criterion can be
114 used in conjunction with the Mises, Johnson-Cook, Hill, and Drucker-Prager
115 plasticity models [26]. More precisely, the ductile damage model that we use
116 and adapt to our need is inspired from this article [27]. It has its justification

117 from the anisotropic behaviour due to the specific orientation of the fiber in-
 118 side the matrix.
 119

120 2.3. Rupture criteria under high pressure

121 The damage initiation criterion in this model is defined in terms of the
 122 effective plastic strain $\bar{\epsilon}_{pl}$. The damage propagate when the scalar ω reaches
 123 1, where ω is defined as:

$$\omega = \int_0^t dt \frac{\dot{\bar{\epsilon}}_{pl}}{\bar{\epsilon}_{pl}^{\bar{\epsilon}r}(\eta, \bar{\epsilon}_{pl})} \quad \text{with} \quad \dot{\bar{\epsilon}}_{pl} = \sqrt{\frac{2}{3} \dot{\epsilon}_{pl} : \dot{\epsilon}_{pl}} \quad (3)$$

124 $\bar{\epsilon}_{pl}$ is defined as the plastic deformation. It increases monotonically as the
 125 state variable ω increases. Here, $\bar{\epsilon}_{pl}^{\bar{\epsilon}r}(\eta, \bar{\epsilon}_{pl})$ is the equivalent plastic strain at
 126 the onset of damage, and η is the triaxiality. $\dot{\bar{\epsilon}}_{pl}$ is the strain rate.
 127 The damage initiation occurs when the equation 3 (satisfies $\omega = 1$). There
 128 here a function of triaxiality and strain-rate to take into account the triaxial-
 129 ity and rate dependent damage found in thermoplastic matrices. Numerical
 130 values used are reported in Table 2.
 131 The damaged stress tensor σ can be expressed as following:

$$\sigma = \bar{\sigma}(1 - D) \quad (4)$$

132 where $\bar{\sigma}$ is the undamaged stress tensor. D , is the damage variable that
 133 increases from 0 to 1. The damage evaluation [26] can be defined using
 134 displacement at failure in two cases: linear form and exponential form. The
 135 governing equation for equivalent plastic displacement is:

$$D = D(u^{pl}) \quad (5)$$

136 Where, u^{pl} is the effective plastic displacement. In this study linear form is
 137 used. The software used in this study for FEM simulation, is ABAQUS. It
 138 offers a damage model which allow to predict the onset and the damage evo-
 139 lution for elasto-plastic materials with anisotropic behavior. The ABAQUS
 140 anisotropic damage model is based on the work of Hashin, Matzenmiller and
 141 Camanho, [20, 19, 21].

142 **3. Material and method**

143 The shape of the RVE is chosen according to the microstructure, which
144 can be an accurate reflection of reality or an idealization [28, 4]. In this study,
145 in the case of unidirectional fibers distributed at the center of the RVE, the
146 body-centered cubic packing is chosen as it is an ideal representation of the
147 composites. The choice of the RVE type is motivated by the microstructure
148 according to these studies [29, 30, 31].

149
150 The numerical definition of each phase of the composite corresponds to
151 the matrix and fiber. In this study, we numerically explored five different
152 polymers for the matrix since they are the most common [32] as shown in
153 table 3. Two types of fibers are used; glass and aramid one. The matrix and
154 fiber which constitute the composite are separately defined in a local coordi-
155 nate system. The material properties to be inserted in the model are defined
156 in terms of engineering constants: Young modulus E and Poisson ratio ν .
157 The mechanical properties of the RVEs are defined within the ABAQUS
158 module named elasticity. This subroutine module uses the previous values
159 of the constant of Young modulus and Poisson ratio to reproduce the elastic
160 properties. We add tabular values (see Table 3) for the elasto-plastic prop-
161 erties of both fiber [33] and matrix. In order to describe the behaviour of
162 the material at a displacement stress, the values of plastic strain according
163 to the yield stress are implemented manually (see next Table 1).

Table 1: Explored range of the input parameters of the plastic behaviour P_s and the yield value Y_s of each matrix used in FEM simulation. Extracted from this reference [34]

PA6		PA12		PP		LDPE		HDPE	
$P_{S(\text{MPa})}$	$Y_S(\%)$	$P_{S(\text{MPa})}$	$Y_S(\%)$	$P_{S(\text{MPa})}$	$Y_S(\%)$	$P_{S(\text{MPa})}$	$Y_S(\%)$	$P_{S(\text{MPa})}$	$Y_S(\%)$
0.1	0	0.16	0.01	0.0	0.0	13.0	0.0	0.1	0.0
5.50	0.16	7.71	0.5	14.38	0.10	13.2	0.01	11.32	0.96
6.42	0.19	14.8	1	17.54	0.20	13.3	0.02	16.91	1.84
7.34	0.22	21.3	1.5	20.34	0.35	13.4	0.1	21.1	3.11
8.26	0.25	26.9	1.99	22.43	0.54	13.5	0.15	23.82	4.82
25.0	0.77	32	2.5	24.70	0.77	13.6	0.35	25.14	6.79
42.4	1.32	36	3.0	26.8	1.14	13.7	1.0	25.55	9.608
53.4	1.70	38.3	3.5	28.8	1.57	13.8	3.0	25.17	13.31
63.9	2.09	39.7	4.0	30.2	2.10			22.70	24.15
73.2	2.49	40.3	4.5	31.1	2.72				
80.7	2.92	40.5	5.0	31.5	3.42				
85.6	3.39			30.1	4.71				
87.4	3.95			30.32	5.98				

164 In this study, the ductile damage subroutine involves the initiation crite-
165 rion with the following variables as presented in Table 2. The overall damage
166 variable is D . The default setting of D depends on whether elements should
167 be deleted or not after reaching the maximum degradation. Output variable
168 proper to ABAQUS is SDEG and it contains the value of D . SDEG is defined
169 as overall scalar stiffness degradation. It is used when damage evolution is
specified:

Table 2: Damage parameters used to implement numerically the ductile damage of the composite RVE [35]

Fracture strain	Stress triaxiality	Strain rate
0.05	-0.333	0.1

171 4. Modeling methodology : Finite element modeling

172 By employing those input parameters and subroutine of elasticity, plas-
173 ticity, ductile damage, and elasto-plastic law, the response of the RVE subb-
174 mitted to different type of sollicitation from tensile traction to compression and
175 shear will be determined. The constitutive modeling approach is used to cre-
176 ate a model that reflects comprehensively the behaviour of the test material.
177 To achieve this goal, FEM simulation will follow the different steps as pre-
178 cised in the next section. This section explains the basic steps of the Python
179 codes. The code is called in ABAQUS/CAE through the menu command
180 File, then the script is run

181 Developing a constitutive models requires to consider the relationship
182 between stress and strain [36, 37]. This relationships gives the most reliable
183 insight window into the mechanics of material behaviour. Therefore, linear
184 elasticity models are the simplest and most common material models in use.
185 They are the key (practical) material models used for solids mechanics and
186 structural analyses. This law has to be generalized into a triaxial stress and
187 strain state where the proportionality constant is a fourth-order tensor. The
188 most general vectorized expression of linear elasticity, implemented in the
189 FEM solvers is given in the following equation:

$$\sigma = D : \epsilon \quad (6)$$

190 Where σ and ϵ are second-order stress and strain tensors, which can be pre-
191 sented in the Voigt contracted notation, and D is a fourth-order tensor called
192 the stiffness tensor. In practice, the set of 81 terms of the D tensor is always
193 reduced out of presence of symmetry. In the following, we consider special
194 cases of isotropic transverse symmetry for the stiffness tensor. This means
195 that the material possesses a plane of symmetry. A longitudinal axis is ori-
196 ented in, the Z -axis as presented in Figure 2(a). All properties in the X
197 and Y -directions will be similar. A fibre reinforced unidirectional composite
198 shows this property with the modulus, in the main fibre direction, E_{33} for the
199 Z or number 3 axis. The properties in the two transverse axes (E_{22} and E_{33} ,
200 for example) will be the same. In short, the unidirectional ply material is
201 stiffer in the direction of the fiber than in the others directions. A symmetry
202 is observed in the longitudinal direction. This model will be expressed in an
203 orthonormal basis as following:

204

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_L} & -\frac{\nu_{LT}}{E_L} & -\frac{\nu_{LT}}{E_L} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & \frac{1}{E_T} & -\frac{\nu_{LT}}{E_L} & 0 & 0 & 0 \\ -\frac{\nu_{LT}}{E_L} & -\frac{\nu_{TT}}{E_T} & \frac{1}{E_T} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{TT})}{E_T} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{LT}} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} \quad (7)$$

205 Where the E_s are the Young modulus and the ν_{LT}, ν_{TT} correspond to
 206 Poisson ratio in the "longi/trans" and "trans/trans" direction. The G_{LT} and
 207 G_{TT} are the shear modulus.

208 4.1. (RVE) modeling of unidirectional composite in ABAQUS

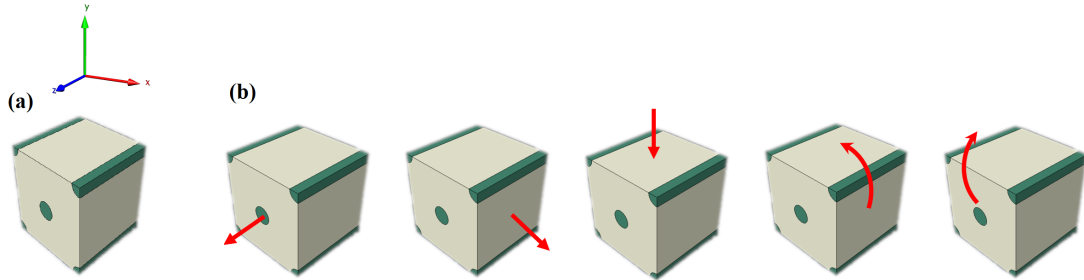


Figure 2: *In (a)* 3D RVE representing in *green* the fiber and in *white* the matrix with the (bcc) symmetry generated by ABAQUS. *In (b)* the 5 sollicitations realized by the homogenisation method from *left to right* (X-tensile, Y-tensile, Z-compression, Shear in plan (XY), shear in plan (YZ)).

209 The unit cell employed in the present study uses a (bcc) symmetry of the
 210 RVE. Aligned reinforcements according to a Z-axis direction are generated at
 211 the center and four corners of the cubic volume as we can see in Figure 2(a).
 212 In order to allow a direct assessment of fiber-matrix effects, 100 different RVE
 213 were generated sequentially. Cylindrical fibers occupy the same positions
 214 and have the same orientations. Aspect ratio and reinforcement volume
 215 fraction were looped over in a range between [35% – 40%] according to the
 216 following formula [38] that relates volume fraction (F_v) to the radius of the
 217 fiber $R_{fiber} = \left(\frac{F_v W H L}{\pi} \right)^{1/3}$. Where W, L, H are the weight, length, and height
 218 of the unit cell. The geometric parameters of the RVE are defined into unite

219 value $W = L = H = 1$ mm as shown in Figure 2(b). In the simulation, we
 220 use a python script to set the main variables that drive numerical simulation
 221 on ABAQUS engine solver. We loop over the fiber properties (E, ν) as well
 222 as the matrix's characteristics and the volume fraction F_v as it is exposed in
 223 the following Table 3:

Table 3: Detailed range values of the input parameters used in the python script: Matrix and fiber properties, Volume fraction percentage, and sollicitation direction.

Matrix name	PA12	PA6	PP	LDPE	HDPE
$(E_{Young}(MPa), \nu)$	(1250, 0.39)	(800, 0.4)	(1550, 0.43)	(1080, 0.45)	(565, 0.46)
Fiber name	glass	aramid			
$(E_{Young}(MPa), \nu)$	(8000, 0.34)	(3500, 0.35)			
Volume fraction	35%	40%			
Sollicitation	Tensile:X, Z	Compression:Y	Shear:XY, YZ		

224 The used value for fibers are taken from the following references: (cited
 225 in foote notes ^{1 2}). In the table 4 , values of the isotropic plastic data for
 226 both glass and aramid fibers are presented.

¹Campus datasheet DUPONT engineering Plastic Zytel®G35HSLRA4BK267-PA66GF35

²Campus datasheet DUPONT engineering Plastic Zytel® 70K20HSL BK284-PA66AF20

Table 4: Explored range of the input parameters of the plastic behaviour of each fiber used in FEM simulation. The datas are taken from the following website [34]

Glass fiber		Aramid fiber	
Plastic strain (MPa)	Yield stress (%)	Plastic strain (MPa)	Yield stress (%)
0	0	0	0
27.1	0.22	26.02	0.54
60.99	0.52	49.28	1.08
78.57	0.68	68.01	1.62
95.32	0.85	81.52	2.17
109.02	1	89.97	2.71
121.29	1.14	95.14	3.25
133.38	1.29	98.62	3.79
145.26	1.45	101.39	4.32
157.62	1.64	103.83	4.86
169.36	1.83	105.34	5.4
180.78	2.05		
191.22	2.31		
200.01	2.6		
204.36	3.01		

227 *4.2. Finite element mesh generation*

228 A graphical illustration of the convergence principle is shown in the Figure
 229 3(a), where a material strain in Z direction S_{33} , is calculated following an
 230 FEM process while changing mesh number (and consequently density) from
 231 a few hundreds of meshes to thousand of meshes.

232 As the mesh density increases, the FEM solution will continue to change
 233 until an optimal solution is reached (**red dashed line**) at that moment there
 234 is very few changes that occur in the predicted parameter (e.g. strain in Z
 235 direction, etc.). The choice of hexahedral mesh is made based on two reasons.
 236 At first it allows to reach more precision on the homogenization of deformation
 237 in comparison with tetrahedral mesh. The second reason is the fact that
 238 the longer time it takes the higher precision in learning will be reached.

239
 240 Therefore, the mesh value fixed is defined in ABAQUS as fine mesh. This
 241 corresponds to: 740 nodes, *ie* 1400 elements. The fibre part is finely meshed

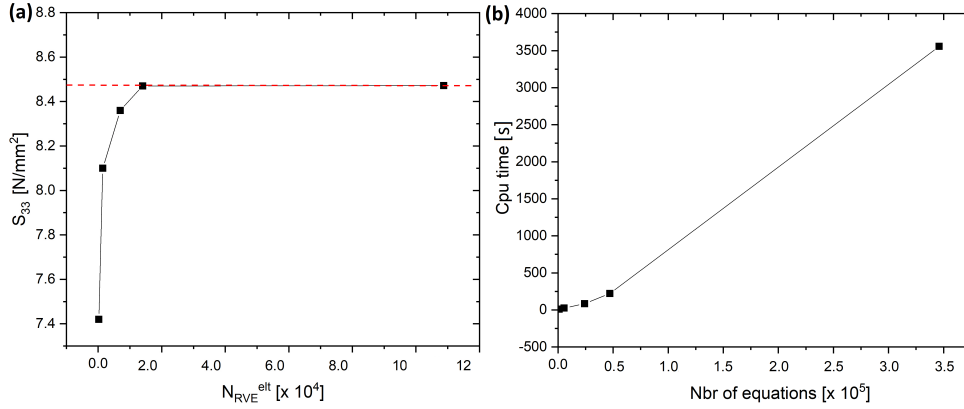


Figure 3: (a) The dependence of FEM solutions on mesh density for strain values S_{33} for an RVE. (b) Evolution of CPU time according to number of equations.

242 as well. The Figure 3 illustrates the meshing result on RVE on both matrix
 243 and fiber parts. The reference point (RP) is settled to indicate the rigid
 244 body reference point. Constraints that were applied to the reference point
 245 are then applied to the entire rigid part. In FEM study, in order to reach
 246 convergence the solution must be mesh size independent. For this reason a
 247 balance between fine mesh and time of calculus 3(b) was fixed (*corresponding*
 248 *to 1400 elements*).

249 4.3. Implementation of periodic boundary conditions:

250 Periodic boundary conditions (PBC) were applied to the RVEs. Those
 251 PBC are used for Homogenization method [39, 40]. First, the resolution of
 252 any FEM problem simulation is dependent on the implemented boundary
 253 conditions [39]. PBCs are widely used since it has been shown in literature
 254 that they can quickly arrive at the convergent solution of an FEM problem
 255 [41, 42]. The PBC stipulates that opposite pairs of edges or surfaces on the
 256 boundary of an RVE should deform identically under a given loading his-
 257 tory [43]. The PBC essentially precludes the constraint of enforcing edges
 258 to remain planar after deformation. In ABAQUS, for most RVEs, boundary
 259 surfaces always appear in parallel pair. The displacement of a pair of oppo-
 260 site parallel boundary surfaces as proposed by Li and Wongsto [4], and the
 261 distortion for each pair of nodes on the parallel boundary surface of the RVE

262 are written according to the following formula:

$$\begin{cases} u_i^A = \bar{\epsilon}_{ik} x_k^A + u_i^* \\ u_i^B = \bar{\epsilon}_{ik} x_k^B + u_i^* \end{cases} \quad (8)$$

263 A and B are the name of parallel surface contacts where i is the i^{th} num-
 264 ber of the pair nodes. the paired nodes of surfaces in ABAQUS are taken
 265 when geometry of study is initially undeformed.

266
 267 To ensure the symmetry of mesh nodes on the periodic plane, three par-
 268 titions according to three symmetry plans $(XY), (YZ), (ZX)$ are made.

269 Here $\bar{\epsilon}_{ik}$ is the average strain of the RVE and u_i^* is the displacement
 270 components of the periodic part. Therefore, it is possible to write the relative
 271 displacement between two nodes as follows [38]:

$$u_i^A - u_i^B = \bar{\epsilon}_{ik} (x_k^A - x_k^B) = \bar{\epsilon}_{ik} \Delta x_k \quad (9)$$

272 Where Δx_k is the constant for each pair of boundary surfaces parallel. The
 273 previous equation is implemented by setting the linear constraint of the
 274 displacement between each pair of nodes on the parallel boundary surface.
 275 These boundary conditions guarantees continuous displacement but also ten-
 276 sile continuity on the parallel surfaces of the RVE. Each node of a given
 277 surface, must correspond to a node on the opposite surface. The six inde-
 278 pendent boundaries are calculated independently for each RVE. In each case,
 279 an applied PBC separately according to the equation 9. The value of $\bar{\epsilon}$ is
 280 given by six cases, and the matrix \bar{C} can be obtained. The average stress
 281 and strain in RVE are calculated by the following formula:

$$\begin{cases} \bar{\sigma}_{ij} = \frac{1}{V} \int_V dV \sigma_{ij} \\ \bar{\epsilon}_{ij} = \frac{1}{V} \int_V dV \epsilon_{ij} \end{cases} \quad (10)$$

282 Where V is the volume of the RVE. The relationship between the stress
 283 matrix for six boundary conditions and stiffness matrix can be written as
 284 follows:

$$\{\bar{\sigma}\} = \bar{C} \{\bar{\epsilon}\} \quad (11)$$

285 Where $\bar{\sigma}_{ij}$ and $\bar{\epsilon}_{ij}$ are the macro stress and strain tensor for each boundary
 286 conditions. \bar{C} is the elasticity response matrix.

287

288 The determination of the mechanical properties of heterogeneous systems
 289 is based on the virtual work principle. This approach uses only the model
 290 variables (nodal forces and displacements) at the retained (or corner nodes)
 291 to compute the work done within the virtual domain. This will be used
 292 subsequently to determine the stresses, strains tensors.

293 In the present ABAQUS simulation work, we consider a 3D RVE window
 294 enclosing a heterogeneous material by boundary edges identified by Ω_k where
 295 $k = [L, T, R, B]$, for left, top, right and bottom. Nodal reaction k forces and
 296 displacements for 3 retained nodes are given as f_i and x_i where $i = 1, 2, 3$ on
 297 retained nodes N1, N2 and N3. Only nodal displacement and force values for
 298 3 retained nodes will be needed to generate the constitutive behaviour (i.e.
 299 the stress strain response) of the heterogeneous material.

300 In the Figure 2 (b) is represented the 5 solicitations that allow to calculated
 301 numerically the \bar{C} matrix.

302 The \bar{C} tensor is the elasticity response of the heterogeneous material
 303 obtained numerically. In details, it dependants on different inputs variable
 304 of the RVE and can be written by explicit argument as follows :

$$\bar{C}(F_v, E_{matrix}, \nu_{matrix}, E_{fiber}, \nu_{fiber}) \quad (12)$$

305 Where F_v is the volume fraction of the fiber inside the matrix. This form
 306 12 with arguments resulting from input parameter related to mechanical
 307 properties of the RVE , reveals the diversity that can be obtained. Each
 308 time one argument is changed a whole new \bar{C} is obtained. This is an other
 309 reason why ML approach is appropriate in this work.

310 4.4. Field output extraction: measures of stress and strain

311 The FEM process is a predictive modeling, which needs the use of com-
 312 putational methods to determine the behaviour of a given material. The
 313 principles of continuum mechanics is to define the response of a material
 314 body. Specifically, the finite deformation of a material body. The numerical
 315 measures of strains and stresses will concludes the practical formulations of
 316 stresses. Such practical stress formulations include: principal stresses, von
 317 Mises stresses, Tresca stresses etc...

318 Numerically, we scripted an editor to select the tensor variable to be included
 319 in a field output request. ABAQUS automatically writes all components of
 320 that variable to the output database during the step. In the present three-
 321 dimensional model, ABAQUS outputs the three displacement components

322 U_1 , U_2 , and U_3 in addition to 36 other measurable variables that are needed
 323 to characterize the damage process. The main important ones are obtained
 324 after a process of feature selection (see next section) and are summed up in
 the following table 5.

Table 5: Output variables used to monitor the ductile damage of the RVE's composite.

S	PE	LE	RF	SDEG	CFailure
stress	plastic strain	logarithmic strain	reaction forces	Stiffness degradation	failure

325

326 4.5. Results of FEM simulation

327 The visualization module of ABAQUS can be used to output SDEG vari-
 328 able as shown in the next Figure 4. It is observed that the degradation is
 329 generally dependant on the kind of solicitation. Here there is three type of
 330 applyed external forces. For the same amount of load for every RVE we can
 331 see that for a tensile in Z direction, degradation field is distributed along the
 332 direction of fiber and inside only the matrix that took all the damage. This
 333 result is quite logical because of the role of the isotropic fiber. Lower damage
 334 in encountered in fiber (center and corners in green colormap). At contrary,
 335 when damage is obtained during tensile X solicitation, which is perpendic-
 336 ular to the fiber direction, we can observe that fibers took all the damage
 337 all along (red and yellow colormap) elsewhere in the matrix low degradation
 338 field appears in green and blue.

339

340 Compressing solicitation along Y direction seems to highly contribute to
 341 the global degradation of the RVE. In fact, it shows high value of degradation
 342 in addition to broad spreading across the volume of the RVE. The degrada-
 343 tion under compression affects more specifically the fiber than the matrix.
 344 Concerning shear solicitation in both (XY) and (YZ) plan, it is showing the
 345 lowest damage on RVE, this is mainly due to the isotropic resistance of the
 346 fiber along the Z direction.

347

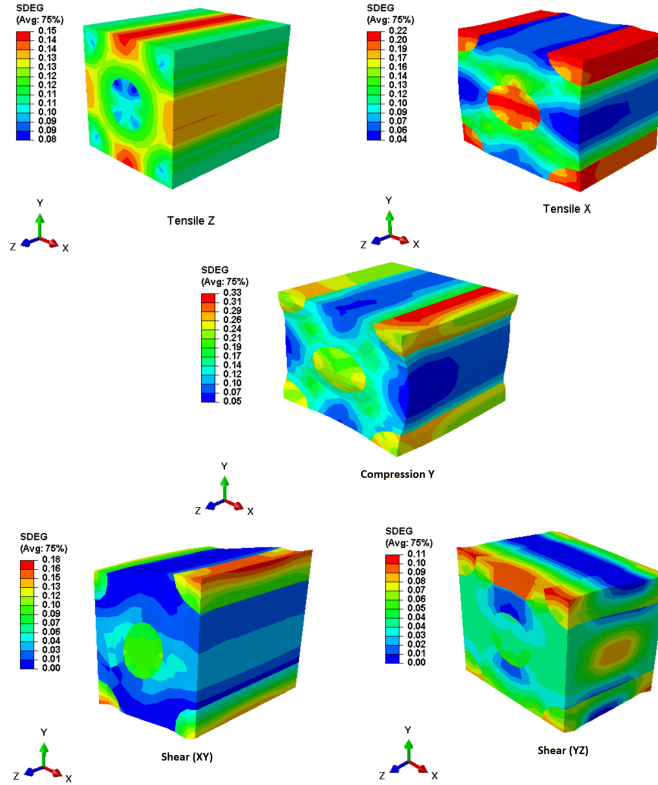


Figure 4: The 5 solicitations effect on degradation realized by the homogenisation method on one RVE ($F_v = 35\%$, $Fiber = aramid$, $Matrix = PP$). *On top* is presented the effect of tensile strength in X, Z direction. *On middle* is the effect of compression along Y direction. Finally, *on bottom* is the shear effect that is represented. The overall scalar stiffness degradation is represented in color map.

348 Obtained results are compared to numerical outcomes presented in this
349 study [35], which, investigates the parameters that controlled damage be-
350 haviour of continuous fiber-reinforced thermoplastic composite. The RVE
351 was submitted to transverse tensile loading and showed good accordance
352 with experimental tensile tests on $[90]_8$ laminated model reported in[44]. For
353 our FEM study, qualitative confirmation can be deduced. In fact, our results
354 are in harmony with outcomes results reported in [45]. When comparing with
355 the four cases treated in the previous study, for transverse tension, transverse
356 compression, in-plane shear there is similarities. in fact, in the epoxy carbon
357 fiber reinforced polymer (CFRP) unidirectional lamina with a random distri-

358 bution of fibers, as for our compression cases, the transverse compression is
 359 the most damaged one. Then in Figure 4 in [45] for transverse compression
 360 the matrix is more affected. This is confirmed in the present simulation by
 361 the strain degradation field that is more spatially extended.

362

363 4.6. Effect of RVE's size variation

364 The effect of RVEs size variation in FEM modeling has been discussed
 365 and some authors [46, 47], have proposed different methodologies to deter-
 366 mine the RVE size. In the present study the choice of the microscopic scale
 367 for the homogenization method bring the question of the effect of the size
 368 on damage evolution [28]. In the present work, was investigated numerically
 369 this effect. Practically, three RVEs sizes have been tested through homog-
 370 enization method with (PBC) conditions. The following sizes of the RVEs
 371 unite celle in micrometer : (8, 12, 16), was implemented on ABAQUS soft-
 372 ware. The stress, strain curve was extracted to compare. In fact, the strain
 373 σ_{33} according to the tensile strength in Z direction was extracted. This ap-
 374 proach uses only nodal forces and displacements at corner nodes to determine
 375 stress and strain of a RVE. The mathematical formulation for deriving the
 376 homogenized (volume-averaged) stresses as in [38] is given as:

$$\langle \sigma_{33} \rangle = \frac{1}{V} [x_1 \otimes f_{N1} + x_2 \otimes f_{N2} + x_3 \otimes f_{N3}] \quad (13)$$

377 Where, f_i and x_i are Nodal reaction forces and displacements for 3 retained
 378 nodes where $i = 1, 2, 3$.

379 In the previous Figure 5 stress values are extracted according to Equation
 380 13 for the different RVE sizes. The classical curve obtained enables to distin-
 381 guish the initial linear domain followed by a narrow plastic domain than the
 382 inflexion of the curve corresponding to degraded stiffness. The superposition
 383 of the three stress strain curves allows the affirmation that the effect of size
 384 variation is not observed. That means that the microscopic approach used
 385 in this study is validated.

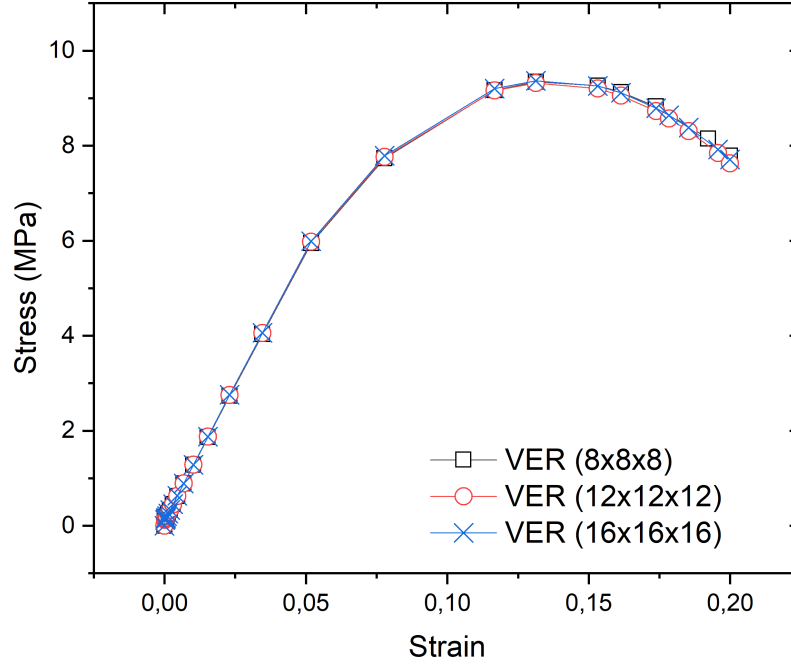


Figure 5: Superposition of Stress-strain curve for three different RVEs size $8\mu m$, $12\mu m$, $16\mu m$.

386 5. Implementation of machine learning model for classification

387 In the last decades, many studies in mechanical material have been using
 388 ML models. It was used to treat data from experiments or from numerical
 389 simulations in a new way that gives a deeper insight and predictions [13].
 390 However, it is important to keep in mind the limitation of (ML) which is the
 391 high dependence on data of quality with implemented models. The recent
 392 advancements in deep learning are intriguing; however, this approach comes
 393 with a significant drawback: the model is essentially a black box. Given that
 394 our study focuses on examining various polymer degradations under pressure,
 395 the imperative to comprehend the underlying processes is a crucial necessity.
 396 Therefore, this disqualifies the use of deep learning models in our context.

397 Figure 6 presents the flowchart of the pipeline:

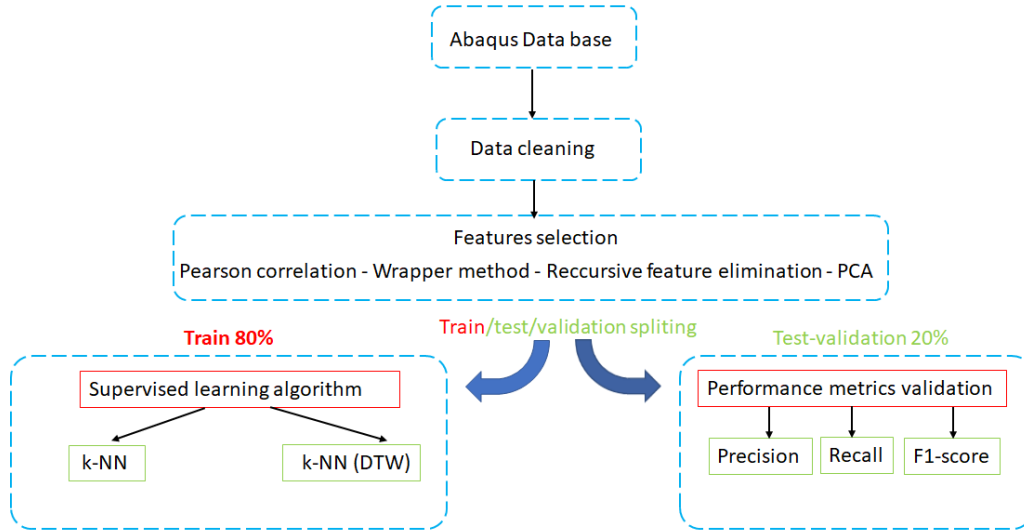


Figure 6: The flowchart of the different steps used for the supervised ML classification. Three major steps are exposed: data cleaning, feature selection, train-test splitting and finally pattern damage classification.

- 398 1. **Database:** to provide valuable data to analyse, ABAQUS is used to
 399 implement and to simulate various RVE's with FEM simulation.
- 400 2. **Data Preprocessing:**
- 401 • **Data Cleaning:** the goal of this step is to consolidate the database
 402 (type of data, type of frame, type of file, how to read and save the
 403 data, etc.). Then the data analysis provides useful tips about the
 404 data, like correlation or multivariate analysis.
- 405 • **Features Selection:** since all the features will not be useful for
 406 the prediction, one have to analyze them to determine the relevant
 407 features. All features have to be independent to the others, must
 408 have a good correlation to the predicted features and must have
 409 a dense data (no NULL inputs in the data).
- 410 3. **Training:** since the goal of our method is a prediction, the data are
 411 splitted into a train-test set respectively 80%-20%. A simple k-NN and
 412 variants algorithm named k-NN with dynamic time warping are used

413 to classify degradation.

414 4. **Validation:** the common metrics for classification are accuracy, preci-
415 sion, recall, F1-score are used to evaluate the accuracy of the model to
416 build classes.

417 5.1. Data Collection and Database Creation

418 Having a dataset of large size significantly enhances the learning process,
419 leading to improved predictions. Utilizing FEM simulation with Python
420 scripting, we generated 100 elasto-plastic models for our study. Parameter-
421 ization of the simulation through loops enables the exploration of a diverse
422 range of Representative Volume Elements (RVEs), making the incorporation
423 of Machine Learning (ML) highly relevant. For this investigation, the data
424 set is made of 200 RVEs generated through FEM simulation in ABAQUS.
425 Each of the 200 RVEs models includes output fields as responses to five dis-
426 tinct solicitations, involving two tensile strains, one compression, and two
427 shear strains (see Figure 2(b)). Each RVE comes with an output database
428 containing 39 output variables (see Table 5) that are columns and 1400 rows
429 that corresponds to the element of discretization.

430 5.2. Features Selection

431 This phase involved the critical process of feature selection, aiming to
432 pinpoint the highly correlated features in the model related to the strain
433 degradation variable. To achieve this, we employed the Pearson correlation
434 matrix, a statistical measure gauging the linear relationship between mul-
435 tiple variables. The resulting correlation matrix provides insights into the
436 interrelationships among the various features in our dataset. The Pearson
437 correlation coefficient ranges from -1 to 1, reflecting the strength and direc-
438 tion of a linear relationship.

439 Examining Figure 7, features with Pearson correlation values greater than
440 0.70 are considered the most positively correlated. To refine our feature se-
441 lection, we applied the Recursive Feature Selection algorithm (RFS) [48],
442 specifically the recursive feature elimination (RFE) variant. This preprocess-
443 ing step is crucial for preventing over-fitting, facilitating faster prediction and
444 training, minimizing storage requirements for both model and dataset, and
445 ultimately producing a more interpretable model.

446 In our study, the RFE algorithm successfully reduced the features to
447 a set of twelve columns, namely: [LE.LE23, LE.LE22, LE.LE12, PE.PE33,

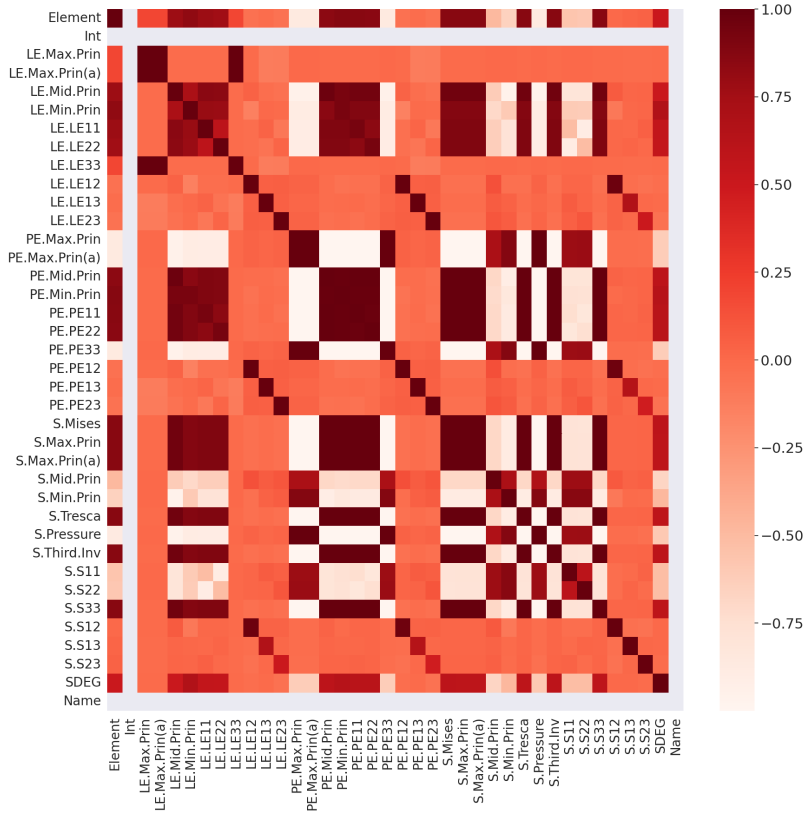


Figure 7: Pearson matrix of correlation of the model features with the SDEG column values.

448 PE.PE11, PE.Min.Prin, PE.PE23, LE.LE33, PE.PE22, LE.Min.Prin, PE.PE12,
 449 SDEG]. This selection, as depicted in Figure 8, represents the most crucial
 450 features obtained from the ABAQUS engine, streamlining the dataset and
 451 enhancing the efficiency and interpretability of our model.

452 5.3. Classification Method

453 The approach throughout using k-NN algorithm had been many times
 454 used to classify and detect the existence of patterns and to rank the most
 455 important features. In the after mentioned study [49] welding defects caused
 456 by laser process was studied to extract the most important features. In the
 457 second study [50], It was possible to extract several feature sets to allow
 458 multi-dimensional classification of damage modes. The presented research
 459 of principal component analysis and pattern recognition methods (k-NN,

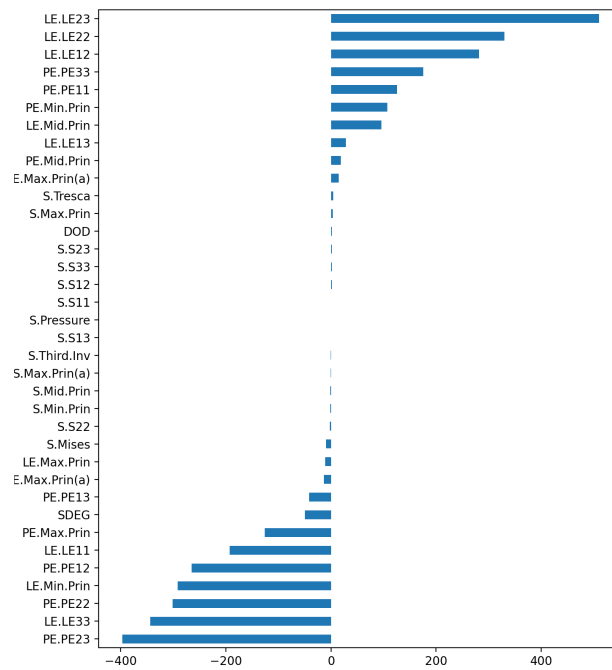


Figure 8: Bar charts plot representing the ordered importance of each feature according the RFE algorithm. Twelve column features are the most important.

460 decision tree, neural networks), has the goal of providing the presence, type,
461 and severity of damage with a high degree of confidence. The last point is this
462 review article [51] that focus on damage assessment on composite materials
463 (detection, quantification and localization). They point out the fact that the
464 success of the machine learning paradigm for damage assessment depends on
465 the representational capability of the discriminative features for the problems
466 of interest.

467 In the subsequent phase of our investigation, we aim to employ a clas-
468 sification approach to distinguish between different degrees of degradation.
469 It exists several type of classification methods mostly based on statistical,
470 probability or pattern recognition. Since the features of the dataset is het-
471 erogeneous with mixed data (numerical and categorical), only non-parametric
472 methods like k-NN [52] fit with the problem. Operating on the assumption
473 that similar objects in the data are close to each other based on a specified
474 metric, the k-NN algorithm is particularly suited for our task.

475 The process of k-NN algorithms is as follows. Given an observation X
476 representing the degradation feature of the RVE we aim to predict, the k-
477 NN algorithm identifies similar objects in the dataset and looks for the k
478 points closest to the given observation. Subsequently, it uses the output
479 variable associated with these nearest neighbors to predict the value of X
480 (i.e., degradation). Noteworthy benefits of k-NN include its simplicity, ease
481 of implementation, and non-parametric nature, eliminating the need for tun-
482 ing multiple hyperparameters. However, its drawback lies in a significant
483 slowdown as the volume of data increases.

484 The k-NN model classifies different RVEs according to the previously
485 defined degradation law. This approach provides valuable output indications
486 about the characteristics of the composite. It enables insights into identifying
487 RVEs similarity in term of behaviors and degradation under the five different
488 solicitations. This information, in turn, allows us to deduce the optimal
489 compositions of fiber and matrix, as well as the best volume fraction F_v .

490 Our approach begins by defining our features (X) and target (y), where
491 the target corresponds to the label of the simulation (referred to as Name
492 in the dataset). Subsequently, we perform a train-test split. Scaling is then
493 executed using the MinMaxScaler function, which fits and transforms the
494 train-test set. The k-NN model is implemented, fitted to our test set, and
495 the training and test sets are scored separately. In this case, the number of
496 k neighbors is set to the default value of 5.

497 *5.4. Performance Metrics*

498 To assess the effectiveness of our model, various performance metrics are
499 employed to evaluate the classification prowess of the k-NN algorithm. The
500 selection of these metrics is pivotal as it provides insight into how well ML
501 algorithms perform and facilitates meaningful comparisons.

502 Currently, our focus lies on the most common metrics utilized for classi-
503 fication problems: accuracy, precision, recall, and F1-score. The Confusion
504 Matrix serves as a valuable tool for visualizing the accuracy of the classifica-
505 tion [53].

506 A **confusion matrix** is a table that describes the performance of a clas-
507 sification model. It presents a summary of the model’s predictions and the
508 actual outcomes. **Accuracy** represents the ratio of correctly predicted in-
509 stances to the total instances in the dataset. **Precision** is the ratio of cor-
510 rectly predicted positive observations to the total predicted positives. It
511 assesses the accuracy of the positive predictions. **Recall** is the ratio of cor-
512 rectly predicted positive observations to the all observations in the actual
513 class. It measures the model’s ability to capture all relevant instances. **F1-**
514 **Score** is the weighted average of precision and recall. It provides a balance
515 between precision and recall.

516 *5.5. Hierarchical Representation*

517 To gain a deeper understanding of the k-NN classification, a valuable ap-
518 proach is to employ an agglomerative clustering algorithm. This algorithm
519 operates in a "bottom-up" manner, initially treating each object as a single-
520 element cluster (leaf). At each step, the algorithm combines the two clusters
521 that are most similar according to Euclidean distance, creating a larger clus-
522 ter (nodes). The agglomerative hierarchical clustering involves the following
523 steps:

- 524 1. **Similarity Matrix:** Calculate the similarity distance between every
525 pair of objects in the dataset.
- 526 2. **Linkage:** Utilize a linkage function to group objects into a hierarchical
527 cluster tree based on the distance information generated in step 1.
528 Objects or clusters in close proximity are linked together using the
529 linkage function (in occurrence, the ward function).
- 530 3. **Cut:** Determine where to cut the hierarchical tree into clusters. This
531 step results in a partition of the data. In this study, the cut is done

532 when the difference of distance between two levels is more than the
 533 total distance of the lower levels.

534 *5.6. Considering Dataset as Time Series and Unsupervised*

535 Dynamic Time Warping (DTW) [54] is an algorithm designed for aligning
 536 temporal sequences and measuring their similarities. It enables local shifts,
 537 contractions, and stretches in temporal sequences and computes a globally
 538 optimal alignment path between two given sequences, subject to certain re-
 539 strictions [55]. The DTW distance, often used to quantify similarity, is the
 540 sum of point-wise distances along the alignment path, expressed as:

$$D(P, Q) = \sum_{(i,j) \in \Pi} d(i, j) \quad (14)$$

541 Here, Π denotes the path between two sequences P and Q , and (i, j)
 542 represents a pair of matched points on the alignment path. The distance
 543 function $d(i, j)$ is the squared Euclidean distance between points i and j
 544 [55]. Figure 9 illustrates the extraction of the optimal path between two
 545 sequences, represented by a red line.

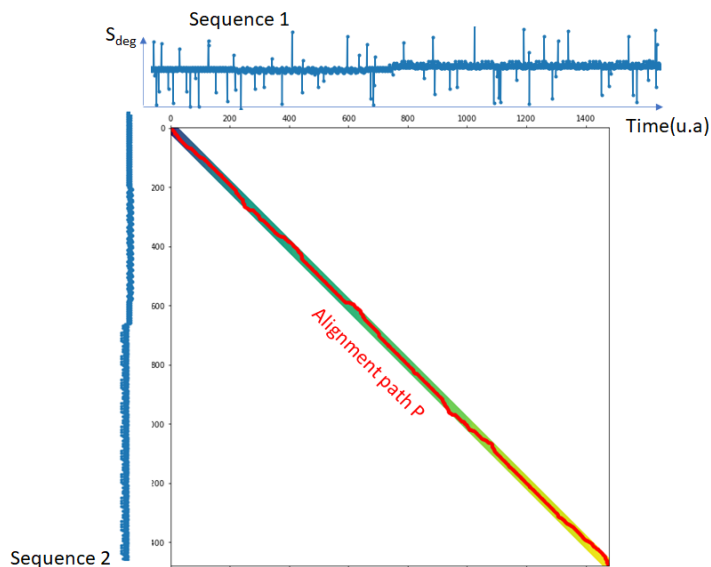


Figure 9: Illustration of the optimal warping path extracted from two sequences of strain degradation SDEG. (*red line*).

546 The focus shifts to training a model to accurately predict the class of
547 degradation using a dataset with labeled time sequences. To address this
548 challenge, the K-Nearest Neighbors algorithm is once again employed, but
549 this time, instead of relying on the classic Euclidean distance, the implemen-
550 tation utilizes the DTW distance. Practically, the Python package *Dtaidistance*
551 [56] is employed, and the k-NN classification leverages a precomputed
552 distance to group time series into different clusters.

553 To demonstrate the quality of results of the DTW distance, we use unsu-
554 pervised hierarchical clustering (thus without the knowledge of the kind of
555 stress, neither the kind of composite polymer).

556 6. Results

557 6.1. *k*-NN Classification Results

558 In Figure 10, the classification outcomes, based on the degradation feature
559 *SDEG*, are illustrated. We utilized the established confusion matrix method
560 from the scikit-learn library [57]. This tabular summary offers an overview
561 of both correct and incorrect predictions made by the k-NN classifier.

562 In this visual representation, each RVE, identified by its unique compo-
563 sition (matrix, fiber, volume fraction), is labeled (represented by numbers).
564 The k-NN classification model’s performance hinges on the comparison be-
565 tween the actual labels of the RVEs and the predicted labels acquired during
566 the learning step. The model’s accuracy is readily discerned by examining
567 the diagonal of the matrix. Dark blue diagonal values indicate proficient
568 model performance, while off-diagonal values suggest lower accuracy in clas-
569 sification. Notably, this distinction is more apparent for labeled models in
570 the intervals [120, 135] and [160, 180]. These cases pertain to two distinct
571 sets of models under shear solicitations in the (*XY*) and (*YZ*) planes, re-
572 spectively. This lower distinguishability in classification is likely influenced
573 by the fiber’s orientation along the *z* direction. Additionally, this reduced
574 classification distinction corresponds particularly to models with the lowest
575 degradation under shear solicitation, as depicted in Figure 4. After the learn-
576 ing phase through training and validation on the validation set, evaluating
577 the model’s performance becomes feasible. The performance metrics used
578 are summarized in Table 6. On average over 200 RVEs, the precision values
579 are quite satisfactory for k-NN classification. A complete Grid search cross
580 validation to fine tune the k-NN parameters has been run. Table 6 summed
581 up the best parameters that had been obtained. This represents 72 models

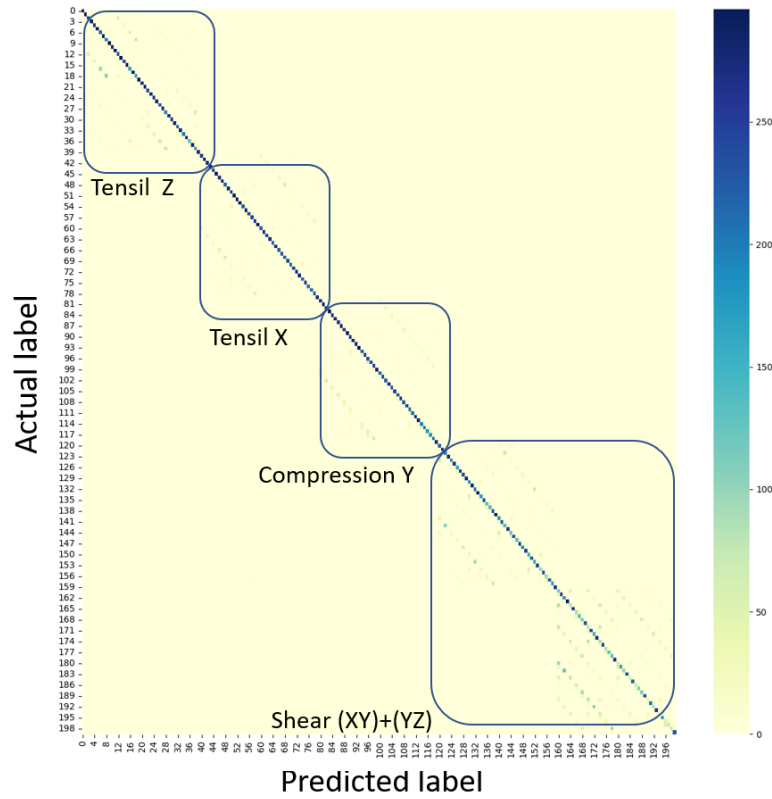


Figure 10: Confusion matrix of the *k*-NN classification for the different 200 RVEs models after data augmentation and recursively features elimination. RVEs corresponding to shear (XY) and (YZ) are less well classified because of the out of diagonal values. Whereas the other solicitations: tensile strength in *X,Z* direction and compression in *Y* direction are fully located on the diagonal.

582 of k-NN to identify the best parameters according to a metric of accuracy.
 583 Those 72 models had been trained then fitted. The confusion matrix used for
 584 visualization reveals that the initial RVEs labels closely approach unity for
 585 precision, recall, and f1-score. This implies that the k-NN algorithm effec-
 586 tively classified the three type of solicitations of tensile stress in (X) and (Z)
 587 and compression stress, whereas the shear solicitation metrics exhibit lower
 values in comparison.

Table 6: Averaged metric values overs the whole FEM models for the classification of the 200 augmented RVEs.

Best model	k	Weights	metric	
K neighbors classifier	4	distance	Manhattan	
Metrics	precision	recall	f1-score	support
accuracy			0.81	57200
macro avg	0.82	0.82	0.81	57200
weighted avg	0.82	0.81	0.81	5730

588

589 6.2. Hierarchical classification from k-NN results

590 Applying a hierarchical clustering on k-NN classification clusters ap-
 591 proach was the principal variation to identify different patterns of degra-
 592 dation similar to this article [58]. In Figure 11(a), the hierarchical repre-
 593 sentation results are presented. The k-NN classification applied on the test
 594 set had predicted four clusters. The dendrogram’s in Y direction named
 595 distance sensitivity revealed that C1 and C2 clusters (*orange color*) are as-
 596 sociated with tensile stress in X and Z directions, while C3 encompasses all
 597 compressions in the Y direction. The most populated cluster, C4, groups
 598 RVEs subjected to shear in both (XY) and (YZ) planes. Figure 11(b) is a
 599 two-dimensional plot of nodes of each RVE affected by degradation. It illus-
 600 trates how clusters were distributed by the k-NN classification. It allowed to
 601 conclude that cluster C4 of RVE submitted to shear has more nodes affected
 602 by degradation in comparison with C2, C3 clusters. At contrary to cluster
 603 C1 (RVEs submitted to tensile Z), the number of affected nodes is more re-
 604 stricted. k-NN punctual analysis enables concluding how far a material will
 605 be damaged based on the cluster assignment

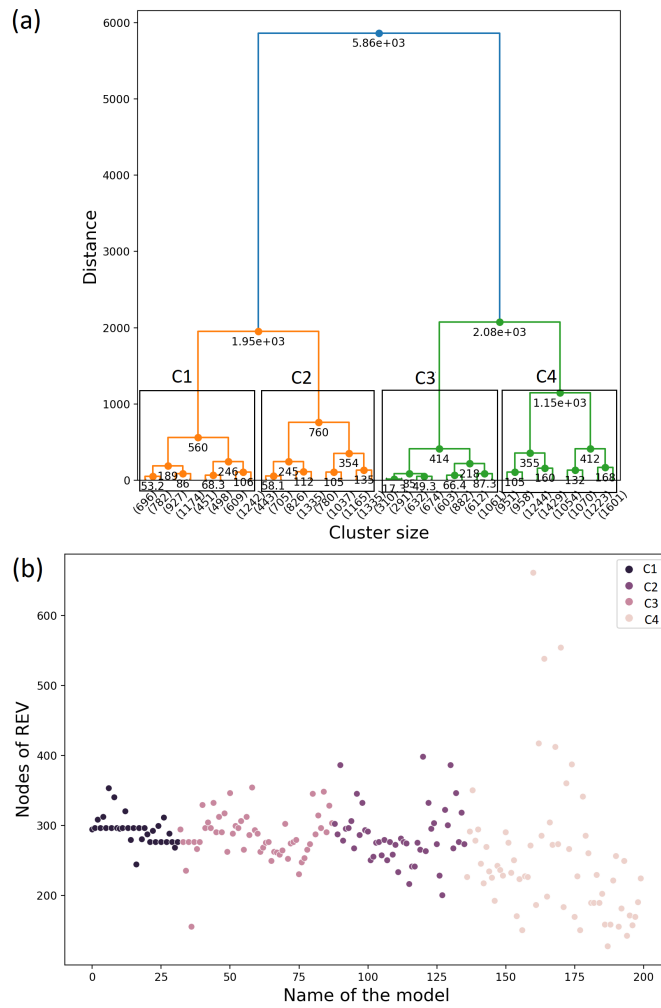


Figure 11: (a) Hierarchical agglomerative clustering of 200 (RVEs) showing 4 main separable clusters. In y direction the euclidean distance report how far the clusters are to one another.(b) Scatter plot graph of the predicted four main clusters resulting from the k -NN classification.

606 Figure 12 compiles box-plots summarizing statistical values of the main
 607 features selected by the RFE algorithm (twelve columns in the dataset). It
 608 depicts feature characteristics through mean values, standard deviation, and
 609 quantiles, grouping values of each feature of the train set according to the
 610 four clusters obtained previously. Consistent patterns emerge for each cluster,
 611 particularly for PE.PE33, PE.PE11, PE.PE22, and SDEG, showcasing
 similar shapes of inter-quartile ranges.

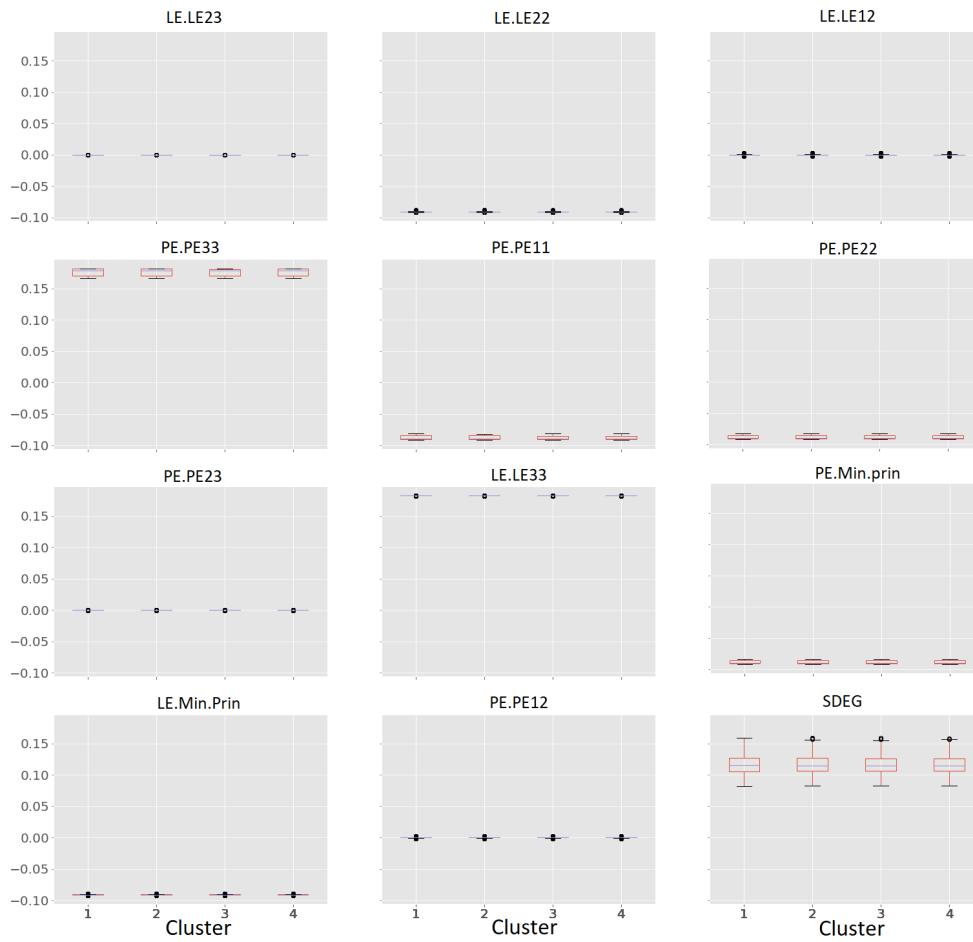


Figure 12: The statistical representation of the extracted features for the four clusters. Boxplot grouping the twelve features selected previously by cluster name C1, C2, C3, C4. Same pattern of boxes according to component of plastic strain (PE11, PE22, PE33) is observed.

612

613 The results of the k-NN model are compared with those of other algo-
 614 rithm models in the next table, where the results are summarizes for random
 615 forest (RF), support vector classifier (SVC), and decision tree (DT) across 6
 616 consecutive trials with different splits (see Table 7).

Table 7: Sum up table of accuracy of classification between models of random forest, decision tree and support vector classifier.

Models	precision	cross validation	CPU time (min)
k-NN	0.82	6	1
Random forest	0.91	6	37
Decision tree	0.86	6	45
Support vector classifier	0.65	6	120

617 From the table 7, it is evident that RF outperforms the other models in
 618 classification accuracy. However, it is worth noting that RFs can be chal-
 619 lenging to interpret and act as a "black box" algorithm. The DT classifier
 620 shows slightly better performance than the k-NN algorithm but with a longer
 621 execution time, and its results are also more challenging to interpret. SVC
 622 is notably less suitable and less efficient compared to the other models.

623 The performance of k-NN is closely tied to the distance metric used.
 624 While the Euclidean distance is simple and effective in some cases, extensive
 625 research [59, 60] suggests that k-NN performance can be enhanced by learning
 626 another distance metric (e.g., Mahalanobis distance). Let's see the results
 627 with DTW distance.

628 *6.3. Results with DTW distance*

629 It is important to notice that the dataset used during this clustering had
 630 no input data or features to inform of the external solicitation applied during
 631 (FEM) simulation.

632 Figure 13(a) illustrates a complete hierarchical tree using the hierarchical
 633 Tree Wrapper class algorithm. The tree provides an order in dissimilarity,
 634 ranging from the cluster with the least dissimilar time series, C1 (represent-
 635 ing tensile strength along the Z -direction), to the most distinct time series
 636 cluster, C3, corresponding to compression in the Y -direction sequences. No-
 637 tably, there is a substantial disparity in the tree distance index value (7.14).

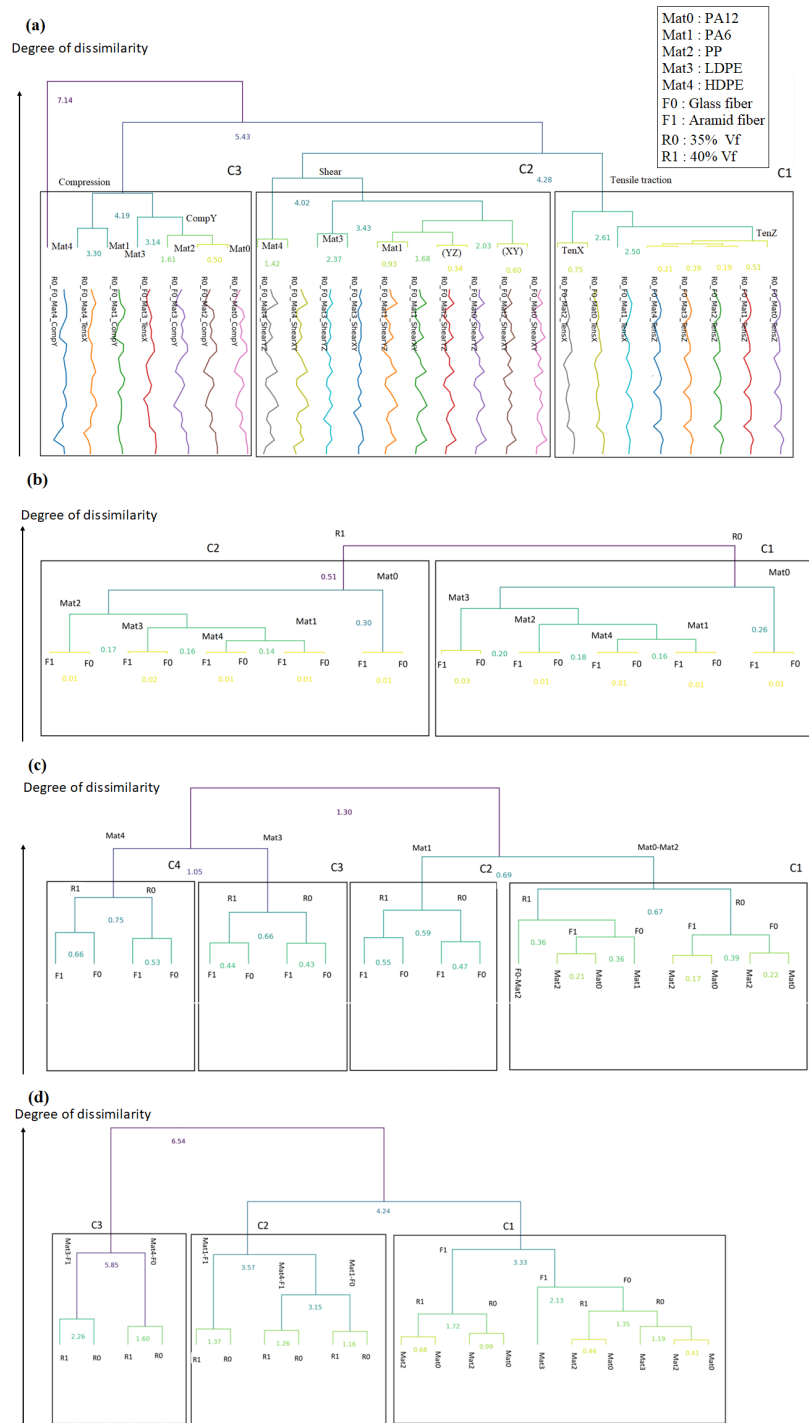


Figure 13: **(a)** Hierarchical clustering tree for 25 different (RVEs) for temporal degradation under different solicitations. Three clusters groups the five different composites were fiber type $F0$ and volume's fraction $R0$ are fixed. **(b)** Hierarchical clustering tree for 20 different (RVEs) temporal degradation during tensile Z solicitation. **(c)** Hierarchical clustering tree for 20 different (RVEs) temporal degradation during shearing (YZ) solicitation. The level of node in tree structure informs about the impact in degradation. **(d)** Hierarchical clustering tree for 20 different (RVEs) temporal degradation during compression solicitation in Y direction. The optimal number of clusters is equal to three.

638 The cluster that aggregates RVEs experiencing degradation due to compression exhibits the highest dissimilarity compared to the other two clusters.
639 This classification based on DTW distance appears to blend sequences of
640 degradation resulting from shear in the (XY) and (YZ) planes. This lack
641 of dissimilarity in shear solicitation was previously observed with the simple
642 k-NN approach.
643

644 6.3.1. Tensile Z

645 In this section, a more systematic study was undertaken to evaluate the
646 DTW k-NN classification. For a fixed external solicitation involving Z -tensile
647 strength, shear in the (YZ) plane, and compression along the Y direction, all
648 other parameters (Matrix type, fiber type, volume fraction) were systematically
649 explored. As reminder, those parameters are not included as inputs in
650 the dataset during the classification.

651 The dendrogram in Figure 13(b) illustrates a hierarchical tree showcasing
652 the shortest distances between 20 RVEs. The present time series depict
653 the evolution of the degradation feature $SDEG$ under Z -tensile strength
654 solicitation over time. Five different materials [$Mat0 - Mat4$], two fibers
655 ($F0, F1$), and two volume fractions ($R0, R1$) are classified, as indicated in
656 (3).

657 Reading the tree from the leaf nodes (graph of the shortest path) to
658 the root nodes reveals that the difference in fibers ($F0 - F1$) has minimal
659 dissimilarity in degradation (index tree path equal to 0.01). However, the
660 nature of the matrix material induces dissimilarities significant enough to
661 categorize each fiber pair ($F0 - F1$) into separate nodes (Mat0, Mat1, Mat2,
662 Mat3, Mat4). The primary feature differentiating in terms of degradation is
663 the volume fraction determined by the radius of the fiber. It is evident that
664 radius $R0$ and $R1$ are the two nodes splitting the tree into two clusters, each
665 with an equal population of RVEs.

666 From this agglomerative hierarchical tree, we can deduce that when applying
667 tensile strength in the direction of the fiber, the radius of the fiber
668 (i.e., volume fraction) plays a crucial role in degradation.

669 6.3.2. Shear in (YZ) plan

670 In the scenario of a constant shear (YZ) solicitation, the Figure 13(c)
671 illustrates how the (DTW) k-NN classification delineates the impact of input
672 features on the final degradation, ranging from the least affected RVEs to

673 the most affected ones. The dendrogram exhibits a distinct tree structure in
674 this case.

675 In Cluster C1, representing minimal dissimilarities, the nature of the
676 matrix material for *Mat0* and *Mat2* (PA12 and PP, respectively) exhibits
677 negligible impact, possibly due to their closely matched Young's modulus
678 values ($E_{PA12} = 1250 \text{ MPa}$ and $E_{PP} = 1550 \text{ MPa}$, see Table 3).

679 However, in Clusters C2, C3, and C4, where the (*XY*) shear induces a
680 more pronounced response, the nature of the matrix material plays a more
681 significant role. Higher dissimilarity levels of 1.05 are observed with matrix
682 materials *Mat1*, *Mat3*, *Mat4*, indicating a distinct tree structure within these
683 clusters.

684 6.3.3. Compression in Y-direction

685 In the context of a fixed compression in the Y direction, a hierarchical tree
686 dendrogram for degradation is obtained through (DTW) k-NN classification
687 applied to 20 RVEs. Figure 13(d) reveals that this mode induces significant
688 dissimilarities in degradation behavior.

689 The tree can be divided into three clusters based on large values of the
690 degree of dissimilarity index: 4.24 between clusters *C1* and *C2*, and 6.54 for
691 cluster *C3*. Cluster *C1* encompasses *Mat0*, *Mat2*, *Mat3*, corresponding to
692 PA12, PP, LDPE, with both glass fiber and aramid glass. Clusters *C2* and
693 *C3* share the same tree structure in terms of nodes.

694 Interestingly, the radius of the fiber (*R0*, *R1*), situated low in the tree,
695 imparts less difference in degradation compared to the nature of the fiber
696 (glass or aramid). For instance, cluster *C3* groups *Mat3* = LDPE when
697 reinforced with Aramid fiber *F1*, and *Mat4* = HDPE with glass fiber.
698 This underscores the significance of the nature of the reinforcement fiber,
699 ultimately altering the material's response to compression in the direction
700 perpendicular to the fiber's orientation.

701 7. Discussions and perspectives

702 7.1. Damage mode detection by k-NN classification

703 In this study, two classification methods, simple k-NN and (DTW) k-NN,
704 were employed to compare the accuracy of classification. The (DTW) classifi-
705 cation provided more nuanced insights through clustering. It became evident
706 that tensile forces in the direction of the *Z* fiber were the least destructive in
707 terms of ductile damage to the matrix-fiber material. The anisotropic nature

708 of the fiber played a protective role, with the radius and nature of the fiber
709 further enhancing this effect. The dendrogram resulting from the (DTW) k-
710 NN classifier for shear in the (YZ) plane solicitation revealed a tree structure
711 similar to the Z-tensile tree. This observation underscores the importance
712 of reinforcing the polymer matrix with multi directional fibers, facilitating
713 strain transfer along the fiber directions. Additionally, the (DTW) k-NN
714 classifier highlighted the higher damage susceptibility of Y-compression, es-
715 pecially in matrices like HDPE and LDPE, which have lower Young modulus
716 values regardless of the fiber’s nature. Lower Young’s modulus values im-
717 ply that these matrices are more compliant, making them more susceptible
718 to deformation and damage under loading conditions. The susceptibility
719 to damage under Y-compression primarily depends on the properties of the
720 matrix material. Even if the reinforcing fibers possess higher strength and
721 stiffness, the overall performance of the composite material can be compro-
722 mised if the matrix material is unable to effectively transfer and distribute
723 loads [61]. Which is the present case.

724 7.2. Comparison of degradation between composite

725 For a more comprehensive understanding, a comparison of the five com-
726 posites in terms of ductile damage was conducted through agglomerative
727 trees. The analysis revealed two distinct sets of matrices.

728 **Set 1:** This group comprises materials with high Young modulus values,
729 namely Mat0 and Mat2 (PA12, PP). During tensile strength in the Z direc-
730 tion, they exhibited the highest dissimilarity, occupying opposite positions
731 in the dendrogram (see Figure 13(b)). For shear (YZ) solicitation in both
732 matrix materials PP and PA12, with either glass or aramid fiber, they have
733 been demonstrating similar degradation, forming cluster C1 13(c)). This can
734 be related to the identical value of shear modulus (400 MPa). The shear
735 modulus provides insights into the material’s behavior when subjected to
736 shear forces perpendicular to the (YZ) plane. Therefore, while the fibers in
737 glass or aramid composites may be relatively insensitive to shear external
738 solicitation, the response of the matrix is important for ensuring the overall
739 mechanical performance of the composite material. In composite materials
740 where the fibers are glass or aramid, it’s generally true [62, 45] that these
741 fibers are relatively insensitive to shear external solicitation compared to the
742 matrix. This is because glass and aramid fibers typically have high tensile
743 strength and stiffness, making them resistant to deformation under shear
744 stress. In such composites, the fibers provide strength and stiffness to the

745 composite structure, while the matrix material serves to transfer loads be-
746 tween the fibers. However, while the fibers themselves may be less sensitive to
747 shear stress, the response of the matrix under shear loading is still important

748

749 **Set 2:** This set includes Mat1, Mat3, and Mat4, representing matrices
750 with lower Young modulus values (800, 1080, 565 MPa, respectively). These
751 matrices occupied central clusters with lower dissimilarities between each
752 other in each dendrogram for tensile, shear, or compression loads.

753 This initial exploration of composite material degradation through the
754 hybrid (FEM-ML) method provides valuable insights for application in real
755 data obtained through experimental means. It is crucial to note that this
756 innovative combination of FEM and ML should be further assessed through
757 an experimental study investigating other types of damage laws.

758 8. Conclusion

759 The study presents a new method that integrates Finite Element Method
760 (FEM) with Machine Learning (ML) to analyze composite degradation. The
761 method uses Dynamical Time Warping (DTW) for KNN classification and vi-
762 sualizes the degree of damage using agglomerative clustering trees. The study
763 suggests that the hybrid FEM-ML approach has potential in predicting dam-
764 age patterns under high external pressure. The primary mode of degradation
765 identified is perpendicular compression to the fiber direction. The study's
766 main contributions are automated multi-class clustering of composite degra-
767 dation and recurrent feature elimination analysis, which identified twelve
768 essential features related to degradation. The k-NN algorithm was imple-
769 mented and achieved an 82% accuracy in classifying composite degradation
770 under various external solicitations. Future research could focus on increas-
771 ing the complexity of numerical simulations, such as incorporating multiple
772 fibers with more intricate loading paths. Furthermore, the hybrid FEM-ML
773 approach could potentially benefit from the implementation of deep learning
774 techniques. This is particularly relevant due to the large number of input
775 parameters involved, which could enhance the model's predictive capabilities
776 and provide valuable insights into composite degradation mechanisms.

777 Data Availability Statements

778 The data that support the findings of this study are available from the
779 authors upon reasonable request and with permission of [De Vinci Research

780 Center].

781 Acknowledgements

782 The authors would like to thank the DeVinci Research center (DVRC)
783 for their implication.

784 Conflict of Interest

785 The authors declare that they have no conflict of interest.

786 References

- 787 [1] M. H. Nemat-Nasser, M. Micromechanics: overall properties of hetero-
788 geneous materials, volume 37, North-Holland, 1993.
- 789 [2] P. Kanout, D. P. Boso, J. Chaboche, B. Schrefler, Multiscale Methods
790 for Composites : A Review (2009) 31–75.
- 791 [3] V. P. Nguyen, M. Stroeven, L. Sluys, Multiscale continuous and discon-
792 tinuous modeling of heterogeneous materials 3 (2011) 229–270.
- 793 [4] S. Li, A. Wongsto, Unit cells for micromechanical analyses of particle-
794 reinforced composites, Mechanics of Materials 36 (2004) 543–572.
- 795 [5] H. Ahmadi, M. Hajikazemi, W. Van Paepegem, Closed-form formu-
796 lae for prediction of homogenized ply-properties and laminate thermo-
797 elastic constants in symmetric laminates containing ply cracks in multi-
798 ple orientations, Composite Structures 241 (2020) 112061.
- 799 [6] K. Daly, T. Roose, Homogenization of two fluid flow in porous me-
800 dia, Proceedings of the Royal Society A: Mathematical, Physical and
801 Engineering Sciences 471 (2015) 20140564–20140564.
- 802 [7] A. Taliercio, R. Coruzzi, Mechanical behaviour of brittle matrix com-
803 posites: A homogenization approach, International Journal of Solids
804 and Structures 36 (1999) 3591–3615.
- 805 [8] G. X. Gu, C. T. Chen, M. J. Buehler, De novo composite design based
806 on machine learning algorithm, Extreme Mechanics Letters 18 (2018)
807 19–28.

- 808 [9] H. Bao, S. Wu, Z. Wu, G. Kang, X. Peng, P. J. Withers, A machine-
809 learning fatigue life prediction approach of additively manufactured met-
810 als, *Engineering Fracture Mechanics* 242 (2021) 107508.
- 811 [10] M. Islam, M. Sohaib, J. Kim, J. M. Kim, Crack classification of a pres-
812 sure vessel using feature selection and deep learning methods, *Sensors*
813 (Switzerland) 18 (2018).
- 814 [11] A. Kaveh, A. Dadras Eslamlou, S. M. Javadi, N. Geran Malek, Machine
815 learning regression approaches for predicting the ultimate buckling load
816 of variable-stiffness composite cylinders 232 (2021) 921–931.
- 817 [12] Regulation no 134 of the economic commission for europe of the united
818 nations (un/ece) — uniform provisions concerning the approval of motor
819 vehicles and their components with regard to the safety-related per-
820 formance of hydrogen-fuelled vehicles (hfcv) [2019/795], 2019. URL:
821 <http://data.europa.eu/eli/reg/2019/795/oj>.
- 822 [13] S. Chibani, F. X. Coudert, Machine learning approaches for the predic-
823 tion of materials properties, *APL Materials* 8 (2020).
- 824 [14] S. Skovsgaard, S. Heide-Jørgensen, Three-dimensional mechanical be-
825 havior of composite with fibre-matrix delamination through homoge-
826 nization of micro-structure, *Composite Structures* 275 (2021) 114–418.
- 827 [15] Z. Ullah, L. Kaczmarczyk, C. J. Pearce, Three-dimensional nonlinear
828 micro meso mechanical response of the fibre-reinforced polymer compos-
829 ites, *Composite Structures* 161 (2016) 204–214.
- 830 [16] F. de Francqueville, P. Gilormini, J. Diani, Representative volume ele-
831 ments for the simulation of isotropic composites highly filled with mono-
832 sized spheres, *International Journal of Solids and Structures* 158 (2019)
833 277–286.
- 834 [17] J. Correia, N. Apetre, A. Arcari, A. De Jesus, M. Muñoz-Calvente,
835 R. Calçada, F. Berto, A. Fernández-Canteli, Generalized probabilis-
836 tic model allowing for various fatigue damage variables, *International*
837 *Journal of Fatigue* 100 (2017) 187–194.
- 838 [18] Z. Hashin, A. Rotem, A fatigue criterion for fiber-reinforced materials,
839 *Journal of Composite Materials* 7 (1973) 448–464.

- 840 [19] Z. Hashin, Failure criteria for unidirectional fiber composites, *Journal*
841 *of Applied Mechanics* 47 (1980) 329–334.
- 842 [20] A. Matzenmiller, J. Lubliner, R. Taylor, A constitutive model for
843 anisotropic damage in fiber-composites, *Mechanics of Materials* 20
844 (1995) 125–152.
- 845 [21] P. Camanho, C. Dávila, Mixed-mode decohesion finite elements for
846 the simulation of delamination in composite materials, NTRS - NASA
847 Technical Reports Server (2002).
- 848 [22] T. Drabek, H. J. Böhm, Damage models for studying ductile matrix
849 failure in composites, *Computational Materials Science* 32 (2005) 329–
850 336.
- 851 [23] A. I. Mel’ker, A. V. Ivanov, Computer modeling of dynamics of damage
852 cumulation in the ductile matrix of a fiber-reinforced composite, *Journal*
853 *of Mechanics of Composite Materials* 23 (1988) 682–685.
- 854 [24] I. Lapczyk, J. A. Hurtado, Progressive damage modeling in fiber-
855 reinforced materials, *Composites Part A: Applied Science and Man-*
856 *ufacturing* 38 (2007) 2333–2341.
- 857 [25] Z. Marciniak, K. Kuczyński, Limit strains in the processes of stretch-
858 forming sheet metal, *International Journal of Mechanical Sciences* 9
859 (1967) 609–620.
- 860 [26] H. Hooputra, H. Gese, H. Dell, H. Werner, A comprehensive failure
861 model for crash worthiness simulation of aluminium extrusions, *Inter-*
862 *national Journal of Crashworthiness* 9 (2004) 449–464.
- 863 [27] R. K. Abu Al-Rub, G. Z. Voyiadjis, On the coupling of anisotropic dam-
864 age and plasticity models for ductile materials, *International Journal of*
865 *Solids and Structures* 40 (2003) 2611–2643.
- 866 [28] K. Terada, M. Hori, T. Kyoya, N. Kikuchi, Simulation of the multi-scale
867 convergence in computational homogenization approaches, *International*
868 *Journal of Solids and Structures* 37 (2000) 2285–2311.
- 869 [29] C. Sun, R. Vaidya, Prediction of composite properties from a represen-
870 tative volume element, *Composites Science and Technology* 56 (1996)
871 171–179.

- 872 [30] M. Arif, N. Saintier, F. Meraghni, J. Fitoussi, Y. Chemisky, G. Robert,
873 Multiscale fatigue damage characterization in short glass fiber reinforced
874 polyamide-66, *Composites Part B: Engineering* 61 (2014) 55–65.
- 875 [31] J. Fitoussi, F. Meraghni, Z. Jendli, G. Hug, D. Baptiste, Experimental
876 methodology for high strain-rates tensile behaviour analysis of polymer
877 matrix composites, *Composites Science and Technology* 65 (2005) 2174–
878 2188.
- 879 [32] H. Fujiwara, H. Ono, K. Ohyama, M. Kasai, F. Kaneko, S. Nishimura,
880 Hydrogen permeation under high pressure conditions and the destruc-
881 tion of exposed polyethylene-property of polymeric materials for high-
882 pressure hydrogen devices, *International Journal of Hydrogen Energy*
883 46 (2021) 11832–11848.
- 884 [33] H. Mehdipour, P. P. Camanho, G. Belingardi, Elasto-plastic constitutive
885 equations for short fiber reinforced polymers, *Composites Part B* 165
886 (2019) 199–214.
- 887 [34] C. mbH, Campus® - a material information system for the plastics
888 industry, 2022. URL: <https://www.campusplastics.com/>.
- 889 [35] D. Pulungan, G. Lubineau, A. Yudhanto, R. Yaldiz, W. Schijve, Ident-
890 ifying design parameters controlling damage behaviors of continuous
891 fiber-reinforced thermoplastic composites using micromechanics as a vir-
892 tual testing tool, *International Journal of Solids and Structures* 117
893 (2017) 177–190.
- 894 [36] A. Ali, M. Hosseini, B. B. Sahari, A review of constitutive models for
895 rubber-like materials, *American Journal of Engineering and Applied*
896 *Sciences* 3 (2010) 232–239.
- 897 [37] T. Beda, An approach for hyperelastic model-building and parameters
898 estimation a review of constitutive models, *European Polymer Journal*
899 50 (2014) 97–108.
- 900 [38] M. Okereke, S. Keates, *Finite element applications: A Practical Guide*
901 *to the FEM Process*, Springer International Publishing, 2018.

- 902 [39] Q.-C. He, Effects of size and boundary conditions on the yield strength
903 of heterogeneous materials, *Journal of the Mechanics and Physics of*
904 *Solids* 49 (2001) 2557–2575.
- 905 [40] V. Kouznetsova, W. Brekelmans, F. Baaijens, An approach to micro-
906 macro modeling of heterogeneous materials, *Computational mechanics*
907 27 (2001) 37–48.
- 908 [41] A. Akpoyomare, M. Okereke, M. Bingley, Virtual testing of composites:
909 Imposing periodic boundary conditions on general finite element meshes,
910 *Composite Structures* 160 (2017) 983–994.
- 911 [42] J. Tyrus, M. Gosz, E. D. Santiago, A local finite element implementation
912 for imposing periodic boundary conditions on composite micromechan-
913 ical modrabekdels, *International Journal of Solids and Structures* 44
914 (2007) 2972–2989.
- 915 [43] I. Gitman, H. Askes, L. Sluys, Representative volume: Existence and
916 size determination, *Engineering Fracture Mechanics* 74 (2007) 2518–
917 2534.
- 918 [44] M. Jerabek, D. Tscharnuter, Z. Major, K. Ravi-Chandar, R. Lang, Mul-
919 ti-axial yield behaviour of polypropylene, *EPJ Web of Conferences* 6
920 (2010).
- 921 [45] L. Wan, Y. Ismail, C. Zhu, P. Zhu, Y. Sheng, J. Liu, D. Yang, Com-
922 putational micromechanics-based prediction of the failure of unidirec-
923 tional composite lamina subjected to transverse and in-plane shear stress
924 states, *Journal of Composite Materials* 54 (2020) 3637–3654.
- 925 [46] D. Savvas, G. Stefanou, M. Papadrakakis, Determination of RVE size
926 for random composites with local volume fraction variation, *Computer*
927 *Methods in Applied Mechanics and Engineering* 305 (2016) 340–358.
- 928 [47] S. M. Mirkhalaf, F. M. Andrade Pires, R. Simoes, Determination of the
929 size of the Representative Volume Element (RVE) for the simulation of
930 heterogeneous polymers at finite strains, *Finite Elements in Analysis*
931 *and Design* 119 (2016) 30–44.
- 932 [48] I. Guyon, A. Elisseeff, An introduction to variable and feature selection,
933 *Journal of machine learning research* 3 (2003) 1157–1182.

- 934 [49] S. H. Lee, J. Mazumder, J. Park, S. Kim, Ranked Feature-Based Laser
935 Material Processing Monitoring and Defect Diagnosis Using k-NN and
936 SVM, *Journal of Manufacturing Processes* 55 (2020) 307–316.
- 937 [50] S. S. Kessler, P. Rani, Pattern recognition for damage character-
938 ization in composite materials, *Collection of Technical Papers -*
939 *AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and*
940 *Materials Conference* 8 (2007) 8507–8517.
- 941 [51] A. Khan, N. Kim, J. K. Shin, H. S. Kim, B. D. Youn, Damage assessment
942 of smart composite structures via machine learning: a review, *JMST*
943 *Advances* 1 (2019) 107–124.
- 944 [52] G. Guo, H. Wang, D. Bell, Y. Bi, K. Greer, KNN model-based ap-
945 proach in classification, 2003. URL: <https://api.semanticscholar.org/CorpusID:1350330>.
- 947 [53] M. Hossin, M. N. Sulaiman, A review on evaluation metrics for data
948 classification evaluations, *International journal of data mining & knowl-*
949 *edge management process* 5 (2015) 1.
- 950 [54] M. Müller, Dynamic time warping, *Information retrieval for music and*
951 *motion* 2 (2007) 69–84. doi:10.1007/978-3-540-74048-3_4.
- 952 [55] S. Kyaagba, Dynamic time warping with time series,
953 2022. URL: https://medium.com/@shachiakyaagba_41915/dynamic-time-warping-with-time-series-1f5c05fb8950.
- 955 [56] W.Meert, Knn dtw, 2022. URL: <https://dtaidistance.readthedocs.io/en/latest/usage/dtw.html>.
- 957 [57] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion,
958 O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, et al.,
959 Scikit-learn: Machine learning in python, *the Journal of machine Learning*
960 *research* 12 (2011) 2825–2830.
- 961 [58] A. Maugeri, M. Barchitta, G. Basile, A. Agodi, Applying a hierarchi-
962 cal clustering on principal components approach to identify different
963 patterns of the SARS-CoV-2 epidemic across Italian regions, *Scientific*
964 *Reports* 11 (2021) 1–9.

- 965 [59] A. Bellet, A. Habrard, M. Sebban, A survey on metric learning for
966 feature vectors and structured data, CoRR abs/1306.6709 (2013). URL:
967 <http://arxiv.org/abs/1306.6709>. arXiv:1306.6709.
- 968 [60] K. Q. Weinberger, L. K. Saul, Distance metric learning for large margin
969 nearest neighbor classification, *Journal of Machine Learning Research*
970 10 (2009) 207–244.
- 971 [61] W. Grellmann, B. Langer, Deformation and Fracture Behaviour of Poly-
972 mer Materials, volume 247, 2017. doi:10.1007/978-3-319-41879-7.
- 973 [62] Fracture behaviour of fibre-reinforced composite materials subjected to
974 shear loading: An experimental and numerical study, *International*
975 *Journal of Lightweight Materials and Manufacture* 6 (2023) 108–116.